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On the existence of local observables in theories with a factorizing *S*-matrix

Gandalf Lechner

Institut für Theoretische Physik, Universität Göttingen, 37077 Göttingen, Germany

E-mail: lechner@physik.uni-goe.de

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Abstract
The scaling limit of the two-dimensional Ising model above the critical temperature is considered as an example for relativistic quantum theories on two-dimensional Minkowski space exhibiting a factorizing *S*-matrix. In this model, a recently proposed criterion for the existence of local quantum field theories with a prescribed factorizing scattering matrix is verified, thereby establishing a new constructive approach to two-dimensional quantum field theory in a particular example. The existence proof is accomplished by analysing the nuclearity properties of certain specific subsets of fermionic Fock spaces, and yields as a byproduct also a verification of the energy nuclearity condition of Buchholz and Wichmann in models of free fermions in four spacetime dimensions.

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1. Introduction

In the last few years a new strategy for the construction of two-dimensional quantum field theories with a factorizing scattering matrix has been developed. At the basis of this approach lies the insight of Schroer and Wiesbrock [25, 27] that factorizing *S*-matrices of massive bosons can be used to define bosonic Wightman fields localized in wedge-shaped regions of two-dimensional Minkowski space by means of the Zamolodchikov algebra [30]. The simple class of two-particle scattering matrices S_2 of a single species of massive particles without bound states has been analysed in [18]. It was shown there that the wedge-local fields on one hand share many properties with a free field, but, on the other hand, lead to non-trivial two-particle scattering states corresponding to S_2. This construction opens a new perspective in the inverse scattering problem in low-dimensional quantum field theory, i.e. the formfactor program [2, 28].

After the construction of the wedge-local fields, a vital issue in this approach is to show that the models so defined also contain observables localized in *bounded* spacetime regions. Whereas the concrete construction of local quantum fields turns out to be very difficult, the existence problem seems to be more easily manageable in the algebraic setting of quantum field theory [14]. There one considers not the wedge-local fields themselves but rather the so-called wedge algebras generated by them. In this framework, the existence of local observables is equivalent to the non-triviality of certain intersections of such algebras [27].

Because of the distinguished geometric action of the modular operators [4, 5, 22] corresponding to wedge algebras and the vacuum, these objects have been studied intensely in local quantum physics. By combining the knowledge scattered in the literature, a convenient sufficient condition for the existence of local observables in wedge-local theories was recently established in [10]. This criterion, known as the modular nuclearity condition [6], has previously been studied in connection with thermodynamical properties of quantum field theories [4, 7].

Given a net $W \longmapsto \mathcal{A}(W)$ of wedge algebras acting on the physical Hilbert space $\mathcal{H}$ with vacuum vector Ω, consider two wedges W_1, W_2, where W_1 contains the causal complement W_2' of W_2, and the double cone region $\mathcal{O} := W_1 \cap W_2$. The content of the modular nuclearity condition is as follows: if the map

$$\Xi : \mathcal{A}(W_2') \longrightarrow \mathcal{H}, \qquad \Xi(A) := \Delta_{W_1}^{1/4} A\Omega, \tag{1.1}$$

is nuclear[1], non-trivial operators localized in $\mathcal{O}$ do exist [10]. Here Δ_{W_1} denotes the modular operator of $(\mathcal{A}(W_1), \Omega)$, which in the models considered acts simply as a boost with an imaginary rapidity parameter [10].

Although this criterion does not solve the task of the explicit construction of local operators, it opens up the possibility of deciding whether such fields exist. Moreover, it provides information about the structure of local algebras determined by them. We are therefore led to the question whether the maps (1.1) are nuclear, for example in the class of S-matrices considered in [18].

As a first step in this direction, we verify in the present paper the modular nuclearity condition in an explicit example of a factorizing theory. The model chosen is fixed by the constant two-particle scattering matrix $S_2 = -1$. It is thus related to the Ising model in the scaling limit, above the critical temperature [3]. The underlying fields are most conveniently represented on an antisymmetric Fock space. Because of this formal analogy with systems of free fermions, it is possible to study nuclearity properties of maps such as (1.1) in a mathematical framework wide enough to cover interaction-free fermionic theories as well. This has the advantage that, as a byproduct of our present investigation, we can also show that the energy nuclearity condition of Buchholz and Wichmann [11] is satisfied in theories describing free fermions. Although this was expected from the thermodynamical interpretation of the energy nuclearity condition, only bosonic theories were shown to satisfy this criterion up to now [9, 11].

This paper is organized as follows. The analysis of nuclearity properties of maps on fermionic Fock space in a general setting is presented in section 2. In section 3, we verify the modular nuclearity condition in the factorizing theory based on the two-particle S-matrix $S_2 = -1$ by applying these results. Some comments about the energy nuclearity condition for free fermions are given in section 4.

[1] See definition 2.1.

2. Nuclear maps on fermionic Fock space

In this section we study the nuclearity properties of certain subsets of antisymmetric Fock space in a general setting. We first explain the mathematical structure involved and then state our main result in proposition 2.1, which yields a criterion for the nuclearity problem mentioned in the introduction and has the advantage that it can be checked on the one-particle space. In section 3, we will apply the results obtained here to the Ising model and show that the one-particle condition is fulfilled in this example.

The mathematical structure needed for our analysis is as follows: let $\mathcal{K}$ be a complex Hilbert space with an antilinear involution $\Gamma = \Gamma^* = \Gamma^{-1}$ acting on it. (In the applications, $\mathcal{K}$ will be realized as a one-particle space of square integrable functions on the upper mass shell and Γ corresponds to complex conjugation in configuration space.) We consider two closed, complex subspaces $\mathcal{L}_\varphi$ and $\mathcal{L}_\pi$ of $\mathcal{K}$ which are invariant under Γ and the real linear subspace defined by

$$\mathcal{L} := (1+\Gamma)\mathcal{L}_\varphi + (1-\Gamma)\mathcal{L}_\pi . \tag{2.1}$$

By second quantization one obtains the antisymmetric Fock space $\mathcal{H}$ over $\mathcal{K}$, the vacuum vector $\Omega \in \mathcal{H}$ and the usual annihilation and creation operators $a(\psi)$ and $a^*(\psi) = a(\psi)^*$, $\psi \in \mathcal{K}$, representing the CAR algebra on $\mathcal{H}$, i.e. ($\psi_1, \psi_2 \in \mathcal{K}$)

$$[a(\psi_1), a(\psi_2)]_+ = 0, \tag{2.2}$$

$$[a(\psi_1), a^*(\psi_2)]_+ = \langle \psi_1, \psi_2 \rangle \cdot 1. \tag{2.3}$$

Here we introduced the notation $[A, B]_\pm = AB \pm BA$ for the (anti-) commutator and $\langle ., . \rangle$ for the scalar product on $\mathcal{K}$. (The scalar product on $\mathcal{H}$ will be denoted by the same symbol.) We adopt the convention that the creation operator $a^*(\psi)$ depends complex linearly on $\psi \in \mathcal{K}$. The CAR relations imply that the annihilation and creation operators are bounded [12]: $\|a(\psi)\| = \|a^*(\psi)\| = \|\psi\|$.

Furthermore, we introduce a fermionic field operator

$$\phi(\psi) := a^*(\psi) + a(\psi), \qquad \psi \in \mathcal{L}, \tag{2.4}$$

as well as the auxiliary fields ($\psi \in \mathcal{K}$)

$$\varphi(\psi) := a^*(\psi) + a(\Gamma\psi), \qquad \pi(\psi) := \mathrm{i}(a^*(\psi) - a(\Gamma\psi)), \tag{2.5}$$

which are related to the time zero Cauchy data of ϕ in the field theoretic context. Note that $\varphi(\psi)^* = \varphi(\Gamma\psi)$, $\pi(\psi)^* = \pi(\Gamma\psi)$ and that $\varphi(\psi_1)$ and $\pi(\psi_2)$ anticommute for arbitrary $\psi_1, \psi_2 \in \mathcal{K}$. For later use we also state

$$a(\Gamma\psi) = \tfrac{1}{2}(\varphi(\psi) + \mathrm{i}\pi(\psi)). \tag{2.6}$$

The field ϕ generates the von Neumann algebra

$$\mathcal{A}(\mathcal{L}) := \{\phi(\psi) : \psi \in \mathcal{L}\}'', \tag{2.7}$$

and we assume that the vacuum vector Ω is seperating for this algebra[2].

The last element needed for our analysis is a densely defined, strictly positive operator X on $\mathcal{K}$, which commutes with the involution Γ. In particular, X is assumed to be invertible. Having in mind the nuclearity conditions mentioned in the introduction, one should think of X as representing one of the following two operators: in connection with the modular nuclearity condition, put $X = \Delta^{1/4}$, where Δ is the modular operator of some von Neumann algebra containing $\mathcal{A}(\mathcal{L})$ with respect to the vacuum vector, and in the context of the energy nuclearity condition, put $X = \mathrm{e}^{-\beta H}$, where $\beta > 0$ is the inverse temperature and H the Hamiltonian of

[2] As in [10, 18], one may equivalently write $\mathcal{A}(\mathcal{L}) = \{\exp(\mathrm{i}\phi(\psi)) : \psi \in \mathcal{L}\}''$ [16].

the theory. As the former example indicates, X is not required to be bounded. We use the same symbol X to denote its second quantization $\bigoplus_{n=0}^{\infty} X^{\otimes n}$ and assume that $\mathcal{A}(\mathcal{L})\Omega$ is contained in its domain.

It is our aim to find sufficient conditions on the real subspace $\mathcal{L}$ and the operator X that imply the nuclearity of the map

$$\Xi_{\mathcal{L}} : \mathcal{A}(\mathcal{L}) \longrightarrow \mathcal{H}, \qquad \Xi_{\mathcal{L}}(A) := XA\Omega. \tag{2.8}$$

For the convenience of the reader, we briefly recall the notion of a nuclear map between two Banach spaces (cf, for example, [23]).

Definition 2.1. *A linear map Ξ between two Banach spaces $\mathcal{A}$ and $\mathcal{H}$ is said to be nuclear if there exists a sequence of linear functionals $\rho_k \in \mathcal{A}^*$, $k \in \mathbb{N}$, and a sequence of vectors $\Psi_k \in \mathcal{H}$, $k \in \mathbb{N}$, such that for all $A \in \mathcal{A}$*

$$\Xi(A) = \sum_{k=1}^{\infty} \rho_k(A) \cdot \Psi_k, \qquad \sum_{k=1}^{\infty} \|\rho_k\|_{\mathcal{A}^*} \|\Psi_k\|_{\mathcal{H}} < \infty. \tag{2.9}$$

The nuclear norm $\|\Xi\|_1$ of such a map is defined as

$$\|\Xi\|_1 := \inf_{\rho,\Psi} \sum_{k=1}^{\infty} \|\rho_k\|_{\mathcal{A}^*} \|\Psi_k\|_{\mathcal{H}}, \tag{2.10}$$

where the infimum is taken with respect to all sequences $\rho_k \in \mathcal{A}^*$, $\Psi_k \in \mathcal{H}$, $k \in \mathbb{N}$, complying with the above conditions.

As Ω seperates $\mathcal{A}(\mathcal{L})$ and X is invertible, the nuclearity of the map $\Xi_{\mathcal{L}}$ is equivalent to the nuclearity of the set

$$\mathscr{N}(X, \mathcal{L}) := \{XA\Omega : A \in \mathcal{A}(\mathcal{L}), \|A\| \leqslant 1\}^{-}, \tag{2.11}$$

which is a subset of $\mathcal{H}$ (the bar indicates closure in the norm topology of $\mathcal{H}$), and the nuclearity index of this set [23] coincides with the nuclear norm of $\Xi_{\mathcal{L}}$. We may thus treat the map (2.8) and the set (2.11) on an equal footing.

Denoting by $E_\varphi, E_\pi \in \mathcal{B}(\mathcal{K})$ the orthogonal projections onto $\mathcal{L}_\varphi, \mathcal{L}_\pi$, respectively, the nuclearity properties of (2.8) are characterized in the following proposition.

Proposition 2.1. *Assume that $E_\varphi X$ and $E_\pi X$ extend to trace class operators on $\mathcal{K}$. Then $\Xi_{\mathcal{L}}$ is a nuclear map, and its nuclear norm is bounded by*

$$\|\Xi_{\mathcal{L}}\|_1 \leqslant \mathrm{e}^{2\|E_\varphi X\|_1} \cdot \mathrm{e}^{2\|E_\pi X\|_1}. \tag{2.12}$$

In comparison with the analogous result for bosons [9, theorem 2.1] one notices two differences: firstly, the conditions on $E_\varphi X$, $E_\pi X$ are relaxed since the bounds $\|E_\varphi X\| < 1$, $\|E_\pi X\| < 1$ on their operator norms are not required here. Secondly, our bound on the nuclearity index is smaller than the corresponding one for bosons, $\det(1 - |E_\varphi X|)^{-2} \cdot \det(1 - |E_\pi X|)^{-2}$, obtained in [9]. This can be seen from the following simple inequality, valid for any non-zero trace class operator T with norm $\|T\| < 1$. The singular values of T are denoted by t_n, repeated according to multiplicity

$$\mathrm{e}^{2\|T\|_1} = \mathrm{e}^{2\sum_{n=1}^{\infty}|t_n|} = \prod_{n=1}^{\infty} (\mathrm{e}^{-|t_n|})^{-2} < \prod_{n=1}^{\infty} (1 - |t_n|)^{-2} = \det(1 - |T|)^{-2}.$$

This result is due to the Pauli principle; it may be understood in analogy with the difference between the partition functions of the non-interacting Bose and Fermi gases in the grand canonical ensemble.

The rest of this section is devoted to the proof of proposition 2.1. To accomplish this proof, we have to amplify the nuclearity of the one-particle operators $E_\varphi X$, $E_\pi X$ to the desired nuclearity statement for $\Xi_{\mathcal{L}}$, which amounts to estimating the 'size' of $\Xi_{\mathcal{L}}(\mathcal{A}(\mathcal{L}))$ as a subset of the fermionic Fock space $\mathcal{H}$. Such an amplification will be achieved by exploiting the structure of underlying CAR algebra in connection with the real linear structure given by Γ.

In a first step, we proceed from $\mathcal{A}(\mathcal{L})$ to the polynomial algebra generated by the field,

$$\mathscr{P}(\mathcal{L}) := \operatorname{span}\{\phi(\psi_1)\cdots\phi(\psi_n) : n \in \mathbb{N}, \psi_i \in \mathcal{L}\}. \tag{2.13}$$

As $\phi(\psi)$ is bounded, $\mathscr{P}(\mathcal{L})$ is a weakly dense subalgebra of $\mathcal{A}(\mathcal{L})$. In view of the closedness of X, we may apply Kaplansky's density theorem and conclude that if the set

$$\mathscr{N}_0(X, \mathcal{L}) := \{XA\Omega : A \in \mathscr{P}(\mathcal{L}), \|A\| \leqslant 1\}^- \tag{2.14}$$

is nuclear, then the larger set (2.11) is nuclear, too, with the same nuclearity index [9]. It is therefore sufficient to study the restriction of $\Xi_{\mathcal{L}}$ to $\mathscr{P}(\mathcal{L})$, and we begin with some comments about this algebra.

The polynomial algebra has the structure of a $\mathbb{Z}_2$-graded $*$-algebra, with even and odd parts $\mathscr{P}^+(\mathcal{L})$ and $\mathscr{P}^-(\mathcal{L})$ given by the linear span of the field monomials of even and odd order, respectively. On $\mathscr{P}(\mathcal{L})$ acts the grading automorphism

$$\gamma(A^+ + A^-) := A^+ - A^-, \qquad A^\pm \in \mathscr{P}(\mathcal{L})^\pm. \tag{2.15}$$

As $\|\gamma\| = 1$ and $A^\pm = \frac{1}{2}(A \pm \gamma(A))$, we conclude $\|A^\pm\| \leqslant \|A\|$.

The following lemma about the interplay of the CAR algebra and $\mathscr{P}(\mathcal{L})$ in connection with the real linear structure of $\mathcal{L}$ is the main technical tool in the proof of proposition 2.1. We will denote the symplectic complement of $\mathcal{L}$ by

$$\mathcal{L}' = \{\psi \in \mathcal{K} : \langle\psi, \xi\rangle = \langle\xi, \psi\rangle \ \forall \xi \in \mathcal{L}\}. \tag{2.16}$$

In preparation recall that an odd derivation on a $\mathbb{Z}_2$-graded algebra $\mathscr{P}$ is a linear map $\delta : \mathscr{P} \to \mathscr{P}$ which satisfies $\delta(\mathscr{P}^\pm) \subset \mathscr{P}^\mp$ and obeys the graded Leibniz rule

$$\delta(A^\pm B) = \delta(A^\pm)B \pm A^\pm\delta(B), \qquad A^\pm \in \mathscr{P}^\pm, \quad B \in \mathscr{P}. \tag{2.17}$$

Lemma 2.1. *For arbitrary $\psi \in \mathcal{K}$, the assignments*

$$\begin{aligned}\delta_\psi^\pm(A) := {}& \tfrac{1}{2}[\varphi((1 \mp \Gamma)\psi) + \mathrm{i}\pi((1 \pm \Gamma)\psi), A^+]_- \\ &+ \tfrac{1}{2}[\varphi((1 \mp \Gamma)\psi) + \mathrm{i}\pi((1 \pm \Gamma)\psi), A^-]_+\end{aligned} \tag{2.18}$$

define two odd derivations on $\mathscr{P}(\mathcal{L})$ which are real linear in ψ. These maps satisfy the bounds

$$\left\|\delta_\psi^+(A^\pm)\right\| \leqslant (\|(1-\Gamma)E_\varphi\psi\|^2 + \|(1+\Gamma)E_\pi\psi\|^2)^{1/2} \cdot \|A^\pm\|, \tag{2.19}$$

$$\left\|\delta_\psi^-(A^\pm)\right\| \leqslant (\|(1+\Gamma)E_\varphi\psi\|^2 + \|(1-\Gamma)E_\pi\psi\|^2)^{1/2} \cdot \|A^\pm\|. \tag{2.20}$$

Moreover, if $\psi \in \mathcal{L}'$,

$$\delta_\psi^+ = 0, \qquad \delta_{\mathrm{i}\psi}^- = 0. \tag{2.21}$$

Proof. The real linearity of $\psi \longmapsto \delta_\psi^\pm$ follows directly from definition (2.18) and the real linearity of φ, π and Γ.

As $\delta_\psi^\pm$ are complex linear maps on $\mathscr{P}(\mathcal{L})$, it suffices to consider their action on field monomials $\phi(\xi_1)\cdots\phi(\xi_n)$, $\xi_1, \ldots, \xi_n \in \mathcal{L}$ to prove the other assertions of the lemma. We also write $\phi_k := \phi(\xi_k)$ and carry out a proof based on induction in the field number n. For $n = 1$, the CAR relations (2.2) and (2.3) imply that

$$\begin{aligned}\delta_\psi^\pm(\phi(\xi)) &= \tfrac{1}{2}\,[\varphi((1 \mp \Gamma)\psi) + \mathrm{i}\pi((1 \pm \Gamma)\psi), \phi(\xi)]_+ \\ &= \tfrac{1}{2}\,(\langle\xi, (1 \mp \Gamma)\psi\rangle \mp \langle(1 \mp \Gamma)\psi, \xi\rangle - \langle\xi, (1 \pm \Gamma)\psi\rangle \pm \langle(1 \pm \Gamma)\psi, \xi\rangle) \cdot 1 \\ &= (\langle\Gamma\psi, \xi\rangle \mp \langle\xi, \Gamma\psi\rangle) \cdot 1.\end{aligned} \tag{2.22}$$

As $\mathcal{L}$ is Γ-invariant, so is $\mathcal{L}'$, and hence $\psi \in \mathcal{L}'$ implies $\delta^+_\psi(\phi(\xi)) = 0$, $\delta^-_{\mathrm{i}\psi}(\phi(\xi)) = 0$. Being a multiple of the identity, $\delta^\pm_\psi(\phi(\xi))$ is contained in $\mathscr{P}^+(\mathcal{L})$ for arbitrary $\psi \in \mathcal{K}$. The step from n to $n+1$ fields is achieved by considering

$$\begin{aligned} [F, \phi_1 \cdots \phi_{2n}]_- &= [F, \phi_1 \cdots \phi_{2n-1}]_+ \cdot \phi_{2n} - \phi_1 \cdots \phi_{2n-1} \cdot [F, \phi_{2n}]_+, \\ [F, \phi_1 \cdots \phi_{2n+1}]_+ &= [F, \phi_1 \cdots \phi_{2n}]_- \cdot \phi_{2n+1} + \phi_1 \cdots \phi_{2n} \cdot [F, \phi_{2n+1}]_+, \end{aligned} \tag{2.23}$$

with $F = \frac{1}{2}(\varphi((1 \mp \Gamma)\psi) + \mathrm{i}\pi((1 \pm \Gamma)\psi))$. It follows from these formulae inductively that $\delta^\pm_\psi$ turn even elements of $\mathscr{P}(\mathcal{L})$ into odd ones and vice versa. Moreover, $\delta^+_\psi = 0$, $\delta^-_{\mathrm{i}\psi} = 0$ for $\psi \in \mathcal{L}'$ because of the corresponding result for $n = 1$. By direct calculation, one can also verify the Leibniz rule (2.17). We have thus shown that $\delta^\pm_\psi$ are odd derivations of $\mathscr{P}(\mathcal{L})$ satisfying (2.21).

To prove the norm estimate (2.19), we first note that

$$\psi' := \left(\tfrac{1}{2}(1+\Gamma)(1 - E_\pi) + \tfrac{1}{2}(1-\Gamma)(1 - E_\varphi)\right)\psi \tag{2.24}$$

is an element of the symplectic complement $\mathcal{L}'$ for arbitrary $\psi \in \mathcal{K}$, as can be easily verified using (2.1). Since $\delta^+_{\psi'} = 0$ and δ^+_ψ is real linear in ψ, we have

$$\begin{aligned} \left\|\delta^+_\psi(A^\pm)\right\| &= \left\|\delta^+_{\psi-\psi'}(A^\pm)\right\| \\ &= \tfrac{1}{2}\|[\varphi((1-\Gamma)E_\varphi\psi) + \mathrm{i}\pi((1+\Gamma)E_\pi\psi), A^\pm]_\mp\| \\ &\leqslant \|\varphi((1-\Gamma)E_\varphi\psi) + \mathrm{i}\pi((1+\Gamma)E_\pi\psi)\| \cdot \|A^\pm\|. \end{aligned} \tag{2.25}$$

To proceed to the estimate (2.19), let $\chi_- := (1-\Gamma)E_\varphi\psi$, $\chi_+ := (1+\Gamma)E_\pi\psi$. As $(\varphi(\chi_-) + \mathrm{i}\pi(\chi_+))^* = -(\varphi(\chi_-) + \mathrm{i}\pi(\chi_+))$ and $\varphi(\chi_-)$ anticommutes with $\pi(\chi_+)$,

$$\|\varphi(\chi_-) + \mathrm{i}\pi(\chi_+)\| = \|\varphi(\chi_-)^2 - \pi(\chi_+)^2\|^{1/2} = (\|\chi_-\|^2 + \|\chi_+\|^2)^{1/2}.$$

Together with (2.25) this implies the claimed norm bound (2.19) for δ^+_ψ. To establish the corresponding inequality (2.20) for δ^-_ψ, consider the vector

$$\psi'' := \left(\tfrac{1}{2}(1-\Gamma)(1 - E_\pi) + \tfrac{1}{2}(1+\Gamma)(1 - E_\varphi)\right)\psi, \tag{2.26}$$

which is contained in $\mathrm{i}\mathcal{L}'$ for any $\psi \in \mathcal{K}$. The norms of $\delta^-_\psi(A^\pm) = \delta^-_{\psi-\psi''}(A^\pm)$ can then be estimated along the same lines as before. □

After these preparations, we now turn to the proof of the nuclearity of $\Xi_\mathcal{L}$ by estimating the size of its image in $\mathcal{H}$. Let $\xi_1, \ldots, \xi_n \in \mathcal{K} \cap \mathrm{dom}(X)$ and $A \in \mathscr{P}(\mathcal{L})$. In view of the second quantization structure of X and the annihilation property of $a(\xi_j)$, we have

$$\begin{aligned} \langle a^*(\Gamma\xi_1)\cdots a^*(\Gamma\xi_n)\Omega, XA^\pm\Omega\rangle &= \langle\Omega, a(X\Gamma\xi_n)\cdots a(X\Gamma\xi_1)A^\pm\Omega\rangle \\ &= \langle\Omega, [a(X\Gamma\xi_n), [\ldots[a(X\Gamma\xi_2), [a(X\Gamma\xi_1), A^\pm]_\mp]_\pm \ldots]_\pm]_\mp\Omega\rangle. \end{aligned} \tag{2.27}$$

From the inside to the outside, commutators and anticommutators are applied alternatingly. We start with a commutator $[a(X\Gamma\xi_1), A^+]_-$ if $A = A^+$ is even and with an anticommutator $[a(X\Gamma\xi_1), A^-]_+$ if $A = A^-$ is odd. Writing the annihilation operator as a linear combination of the auxiliary fields (2.6) and recalling that X commutes with Γ, one notes that the innermost (anti-) commutator is

$$[a(X\Gamma\xi_1), A^\pm]_\mp = \tfrac{1}{2}\left(\delta^+_{X\xi_1} + \delta^-_{X\xi_1}\right)(A^\pm). \tag{2.28}$$

Making use of this equality for all the n (anti-)commutators, it becomes apparent that (2.27) can be rewritten as

$$\langle a^*(\Gamma\xi_1)\cdots a^*(\Gamma\xi_n)\Omega, XA^\pm\Omega\rangle = 2^{-n}\left\langle\Omega, \left(\left(\delta^+_{X\xi_n} + \delta^-_{X\xi_n}\right)\cdots\left(\delta^+_{X\xi_1} + \delta^-_{X\xi_1}\right)(A^\pm)\right)\Omega\right\rangle. \tag{2.29}$$

According to the assumptions of proposition 2.1,

$$T_\varphi := E_\varphi X, \qquad T_\pi := E_\pi X \tag{2.30}$$

are trace class operators on $\mathcal{K}$. Taking into account that $\delta^{\pm}_{X\xi_j}$ are odd derivations on $\mathscr{P}(\mathcal{L})$, an application of the bounds (2.19), (2.20) to (2.29) yields

$$|\langle a^*(\Gamma\xi_1)\cdots a^*(\Gamma\xi_n)\Omega,\, XA^{\pm}\Omega\rangle| \leqslant 2^{-n}\prod_{j=1}^{n}((\|(1-\Gamma)T_\varphi\xi_j\|^2+\|(1+\Gamma)T_\pi\xi_j\|^2)^{1/2}$$
$$+(\|(1+\Gamma)T_\varphi\xi_j\|^2+\|(1-\Gamma)T_\pi\xi_j\|^2)^{1/2})\cdot\|A^{\pm}\|. \tag{2.31}$$

Following [11, 9] we now consider the positive operator

$$T := (|T_\varphi|^2+|T_\pi|^2)^{1/2} \tag{2.32}$$

which is in the trace class, too, satisfies $\|T\|_1 \leqslant \|T_\varphi\|_1+\|T_\pi\|_1$ [17] and commutes with Γ since T_φ and T_π do. As $T^2 \geqslant |T_\varphi|^2$, $T^2 \geqslant |T_\pi|^2$,

$$\left\|\tfrac{1}{2}(1\mp\Gamma)T_\varphi\xi_j\right\|^2+\left\|\tfrac{1}{2}(1\pm\Gamma)T_\pi\xi_j\right\|^2 \leqslant \left\|\tfrac{1}{2}(1\mp\Gamma)T\xi_j\right\|^2+\left\|\tfrac{1}{2}(1\pm\Gamma)T\xi_j\right\|^2=\|T\xi_j\|^2.$$

In terms of T, we thus arrive at the estimate

$$|\langle a^*(\Gamma\xi_1)\cdots a^*(\Gamma\xi_n)\Omega,\, XA^{\pm}\Omega\rangle| \leqslant 2^n\|A^{\pm}\|\cdot\prod_{j=1}^{n}\|T\xi_j\|. \tag{2.33}$$

Although this bound was derived for $\xi_1,\ldots,\xi_n \in \mathcal{K}\cap\mathrm{dom}(X)$ only, it holds for arbitrary $\xi_1,\ldots,\xi_n \in \mathcal{K}$ since $\mathcal{K}\cap\mathrm{dom}(X)\subset\mathcal{K}$ is dense and the left- and right-hand sides of (2.33) are continuous in the ξ_j. With the estimate (2.33), we are now able to give a bound on the nuclearity index of the set (2.11).

The positive trace class operator T acts on $\psi \in \mathcal{K}$ as $T\psi = \sum_{k=1}^{\infty} t_k\langle b_k,\psi\rangle b_k$, where b_k, $k\in\mathbb{N}$, is an orthonormal basis of $\mathcal{K}$ and t_k are the (positive) eigenvalues of T, repeated according to multiplicity, i.e. $\sum_{k=1}^{\infty} t_k = \|T\|_1 < \infty$. Moreover, since Γ and T commute, we may choose the basis vectors b_k to be eigenvectors of Γ as well. As a consequence of the Pauli principle, the vectors

$$b_{\mathbf{k}} := a^*\big(\Gamma b_{k_1}\big)\cdots a^*\big(\Gamma b_{k_n}\big)\Omega = \pm a^*\big(b_{k_1}\big)\cdots a^*\big(b_{k_n}\big)\Omega \tag{2.34}$$

form an orthonormal basis of the totally antisymmetric subspace of $\mathcal{K}^{\otimes n}$ (the fermionic n-particle space) if the multi-index $\mathbf{k} := (k_1,\ldots,k_n)$ varies over $k_1<k_2<\cdots<k_n$, $k_1,\ldots,k_n\in\mathbb{N}$.

Note that $XA\Omega$ has even (odd) particle number if $A\in\mathscr{P}(\mathcal{L})$ is even (odd). By the Fock structure of $\mathcal{H}$, we have for each $\Xi_{\mathcal{L}}(A) = XA\Omega \in \mathscr{N}(X,\mathcal{L})$ the decomposition

$$\Xi_{\mathcal{L}}(A) = \sum_{n=0}^{\infty}\sum_{k_1<\cdots<k_{2n}}\langle b_{\mathbf{k}}, XA^+\Omega\rangle\cdot b_{\mathbf{k}} + \sum_{n=0}^{\infty}\sum_{k_1<\cdots<k_{2n+1}}\langle b_{\mathbf{k}}, XA^-\Omega\rangle\cdot b_{\mathbf{k}}, \tag{2.35}$$

as an example for a representation of type (2.9) of $\Xi_{\mathcal{L}}$. As $\|b_{\mathbf{k}}\| = 1$ for all $k_1,\ldots,k_n\in\mathbb{N}$, and $\|A^{\pm}\|\leqslant\|A\|$, the sum of the expansion coefficients can be estimated with the help of (2.33) as follows:

$$\sum_{n=0}^{\infty}\left(\sum_{1\leqslant k_1<\cdots<k_{2n}}|\langle b_{\mathbf{k}}, XA^+\Omega\rangle| + \sum_{1\leqslant k_1<\cdots<k_{2n+1}}|\langle b_{\mathbf{k}}, XA^-\Omega\rangle|\right)$$
$$\leqslant \|A^+\|\sum_{n=0}^{\infty}2^{2n}\sum_{1\leqslant k_1<\cdots<k_{2n}}\prod_{j=1}^{2n}\left\|Tb_{k_j}\right\| + \|A^-\|\sum_{n=0}^{\infty}2^{2n+1}\sum_{1\leqslant k_1<\cdots<k_{2n+1}}\prod_{j=1}^{2n+1}\left\|Tb_{k_j}\right\|$$
$$\leqslant \|A\|\cdot\sum_{n=0}^{\infty}\sum_{1\leqslant k_1<\cdots<k_n}\prod_{j=1}^{n}2t_{k_j}. \tag{2.36}$$

According to (2.10), the sum (2.36) provides an upper bound for the nuclear norm of $\Xi_{\mathcal{L}}$. To compute this sum, note that (2.36) is nothing else but the partition function of the ideal Fermi gas with Hamiltonian $e^{-\beta H} = 2T$ and zero chemical potential in the grand canonical ensemble. This leads to the estimate (cf, for example, [12])

$$\|\Xi_{\mathcal{L}}\|_1 \leqslant \sum_{n=0}^{\infty} \sum_{1\leqslant k_1<\cdots<k_n} \prod_{j=1}^{n} 2t_{k_j} = \prod_{j=1}^{\infty}(1+2t_j) = \det(1+2T). \tag{2.37}$$

As $\det(1+2T) \leqslant \exp(2\|T\|_1) < \infty$, the nuclearity of $\Xi_{\mathcal{L}}$ follows. Taking into account

$$\|T\|_1 \leqslant \|T_\varphi\|_1 + \|T_\pi\|_1 = \|E_\varphi X\|_1 + \|E_\pi X\|_1, \tag{2.38}$$

we also obtain the bound (2.12) given in the proposition.

3. Application to the Ising model

The two-dimensional Ising model is a lattice model of $\mathbb{Z}_2$-spins with nearest neighbour interaction which is known to undergo a second-order phase transition at some critical temperature. For detailed information on the Ising model and a guide to the literature, see [20]. We point out here only that the correlation functions have been derived in [21, 29], and later by a different method in [24]. The field theory obtained by taking the scaling limit [26] of the Ising model can be described by a Majorana fermion and is known to have a factorizing S-matrix, which is given by the two-particle scattering matrix $S_2 = -1$ [3]. Hence the full S-matrix is

$$S = (-1)^{N(N-1)/2}, \tag{3.1}$$

where N denotes the particle number operator. The corresponding formfactors have been obtained in [3], and in [1] it was shown how to rederive the correlation functions from the formfactors.

We consider here the field theory with the S-matrix (3.1) as a particular simple example of the class of theories considered in [18], and will apply proposition 2.1 to obtain an existence proof for local observables in this model. Besides this possibility of testing our approach to the inverse scattering problem in a first non-trivial example, the point of view taken here gives us the opportunity to add some information to the well-studied Ising model, as no rigorous proof of the Wightman axioms for the n-point functions is known to us.

As the modular nuclearity condition is only a sufficient criterion for the existence of local operators, it is an important test if it is satisfied here. By proposition 2.1, the verification of this condition can be simplified to a problem on the one-particle space.

We begin by briefly recalling the structure of the models under consideration in a manner adapted to the discussion in section 2. For a more thorough treatment, see [18, 10]. The net structure of the algebras of observables localized in wedges W arises from a net of symplectic subspaces of the one-particle space by a 'second quantization' procedure. Recall that in two dimensions, all wedges are translates of the right wedge

$$W_R := \{x \in \mathbb{R}^2 : x_1 > |x_0|\} \tag{3.2}$$

or of its causal complement $W_L = -W_R$.

The one-particle Hilbert space $\mathcal{K}$ can be identified with $L^2(\mathbb{R}, d\theta)$ by using the rapidity parametrization of the upper mass shell, $p(\theta) := m(\cosh\theta, \sinh\theta)$. On $\mathcal{K}$ we have a unitary positive energy represenation U of the proper orthochronous Poincaré group defined as follows: let $B(\lambda)$ denote a proper Lorentz transformation with rapidity λ and $x \in \mathbb{R}^2$ a translation. We set

$$(U(x, B(\lambda))\psi)(\theta) := e^{ip(\theta)x} \cdot \psi(\theta - \lambda). \tag{3.3}$$

We furthermore introduce the notation ω for the one-particle Hamiltonian which acts on a dense domain in $\mathcal{K}$ by multiplication with $m\cosh\theta$. With the help of the involution

$$(\Gamma\psi)(\theta) := \overline{\psi(-\theta)} \tag{3.4}$$

and the auxiliary, Γ-invariant spaces

$$\mathcal{L}_\pm := \{\theta \mapsto \widetilde{f}(m\sinh\theta) : f \in \mathscr{S}(\mathbb{R}_\pm)\}, \tag{3.5}$$

one defines

$$\mathcal{L}_\varphi(W_L + x) := \{U(x)\mathcal{L}_-\}^-, \qquad \mathcal{L}_\pi(W_L + x) := \{U(x)\omega\mathcal{L}_-\}^-, \tag{3.6}$$

$$\mathcal{L}_\varphi(W_R + x) := \{U(x)\mathcal{L}_+\}^-, \qquad \mathcal{L}_\pi(W_R + x) := \{U(x)\omega\mathcal{L}_+\}^-, \tag{3.7}$$

and the real linear subspaces $\mathcal{L}(W) \subset \mathcal{K}$ in accordance with the procedure in section 2 as

$$\mathcal{L}(W) := (1+\Gamma)\mathcal{L}_\varphi(W) + (1-\Gamma)\mathcal{L}_\pi(W). \tag{3.8}$$

The full Hilbert space $\mathcal{H}$ of this model is the fermionic Fock space over $\mathcal{K}$. This is a special case in the class of factorizing S-matrices considered in [18], where one has a representation of Zamolodchikov's algebra

$$z^\dagger(\theta_1)z^\dagger(\theta_2) = S_2(\theta_1 - \theta_2)z^\dagger(\theta_2)z^\dagger(\theta_1), \tag{3.9}$$

$$z(\theta_1)z^\dagger(\theta_2) = S_2(\theta_2 - \theta_1)z^\dagger(\theta_2)z(\theta_1) + \delta(\theta_1 - \theta_2)\cdot 1, \tag{3.10}$$

by operator-valued distributions $z(\theta), z^\dagger(\theta)$ acting on a Fock space with an S_2-dependent symmetry structure [19]. The so-called scattering function S_2 appearing here is closely related to the two-particle S-matrix.

As the S-matrix (3.1) corresponds to the scattering function $S_2(\theta) = -1$, the Hilbert space $\mathcal{H}$ in this case is the fermionic Fock space over $\mathcal{K}$ and one may define annihilation and creation operators representing the CAR algebra (2.2), (2.3) as

$$a^*(\psi) := z^\dagger(\psi) = \int \mathrm{d}\theta\, \psi(\theta)z^\dagger(\theta), \qquad a(\psi) := z(\overline{\psi}) = \int \mathrm{d}\theta\, \overline{\psi(\theta)}z(\theta) \qquad \psi \in \mathcal{K}.$$

Furthermore, the wedge-local field operator considered in [18] has the same form as the field $\phi(\psi)$ defined in (2.4). Note that we do *not* deal here with a free fermionic field, but rather with a Bose field represented on an auxiliary antisymmetric Fock space. As a matter of fact, all factorizing models considered in [18] have bosonic scattering states. By means of ϕ, one can construct a wedge-dual net of von Neumann algebras from the subspaces $\mathcal{L}(W)$ as

$$\mathcal{A}(W_L + x) := \{\phi(\psi) : \psi \in \mathcal{L}(W_L + x)\}'', \qquad \mathcal{A}(W_R + x) := \mathcal{A}(W_L + x)'. \tag{3.11}$$

This net is covariant with respect to (the second quantization of) U, and the Fock vacuum vector $\Omega \in \mathcal{H}$ is cyclic and seperating for each algebra $\mathcal{A}(W)$. Moreover, the modular operators of these algebras with respect to Ω are known to act geometrically correct, i.e. as expected from the Bisognano–Wichmann theorem [10].

In the above construction, we distinguished W_L as reference wedge by generating the algebras associated with left wedges $W_L + x$ by the fields $\phi(\psi)$, $\psi \in \mathcal{L}(W_L + x)$, and defined the algebras associated with right wedges as the corresponding commutants. Another possible definition of the wedge algebras, which distinguishes W_R instead of W_L as reference wedge, is given by interchanging W_L and W_R in (3.11). Note that these two definitions do *not* coincide. With the convention (3.11) used here, $\mathcal{A}(W_R + x)$ is not generated by $\phi(\psi)$, but by a second, different field $\phi'(\psi) = S\phi(\psi)S^*$, lying relatively wedge-local to ϕ [18], i.e.

$$\mathcal{A}(W_R + x) = \{S\phi(\psi)S^* : \psi \in \mathcal{L}(W_R + x)\}''. \tag{3.12}$$

But since S (3.1) commutes with the translations, the two nets arising from distinguishing either W_L or W_R as reference wedge are unitarily equivalent. Therefore we can adopt the convention (3.11) without loss of generality.

To analyse the content of local observables of the net (3.11) of wedge algebras, we consider a double cone $\mathcal{O}_x := W_L \cap (W_R + x)$, $x \in W_L$, and the algebra

$$\mathcal{A}(\mathcal{O}_x) := \mathcal{A}(W_L) \cap \mathcal{A}(W_R + x). \tag{3.13}$$

According to the modular nuclearity condition, the existence of observables localized in $\mathcal{O}_x$ is ensured if the map

$$\Xi(x) \colon \mathcal{A}(W_L) \longrightarrow \mathcal{H}, \qquad \Xi(x)(A) := \Delta^{1/4} U(x) A\Omega, \tag{3.14}$$

is nuclear, where Δ denotes the modular operator of $(\mathcal{A}(W_L), \Omega)$. Comparing with section 2, we see that $\Xi(x)$ has the form of the map $\Xi_{\mathcal{L}}$ considered there with $\mathcal{L} = \mathcal{L}(W_L + x)$ and $X = \Delta^{1/4}$. Indeed, $\Delta^{1/4}$ is closed, strictly positive, and is the seond quantization of its restriction to the one-particle space. One-particle states $\psi \in \mathrm{dom}\,\Delta^{1/4}$ have wavefunctions admitting an analytic continuation to the strip $S\big(0, \frac{\pi}{2}\big) = \big\{\theta \in \mathbb{C} : 0 < \mathrm{Im}(\theta) < \frac{\pi}{2}\big\}$ with continuous boundary values, and one has in particular

$$(\Delta^{1/4}\psi)(\theta) = \psi\left(\theta + \frac{\mathrm{i}\pi}{2}\right). \tag{3.15}$$

Consequently, $\Delta^{1/4}$ commmutes with Γ. We therefore may apply proposition 2.1 to deduce the nuclearity of the map (3.14) from the nuclearity of the one-particle operators

$$T_\varphi(x) := \Delta^{1/4} E_\varphi(W_L + x), \qquad T_\pi(x) := \Delta^{1/4} E_\pi(W_L + x), \qquad x \in W_L, \tag{3.16}$$

where $E_\varphi(W_L + x), E_\pi(W_L + x) \in \mathcal{B}(\mathcal{K})$ denote the orthogonal projections on the subspaces $\mathcal{L}_\varphi(W_L + x)$ and $\mathcal{L}_\pi(W_L + x)$, respectively. But the operators (3.16) were already shown to be of trace class on $L^2(\mathbb{R}, \mathrm{d}\theta)$ in [10]. We have thus verified the modular nuclearity condition in this model, thereby finishing the construction of the corresponding net of local algebras (3.13).

To summarize, the map $\mathcal{O} \longmapsto \mathcal{A}(\mathcal{O})$ defined by (3.13) and (3.11) is an isotonous, local net of von Neumann algebras which is covariant with respect to the action of the representation U because of the corresponding properties of the net of wedge algebras [18]. Furthermore, we established here the following proposition.

Proposition 3.1. *In the model theory with S-matrix* $S = (-1)^{N(N-1)/2}$*, the maps* $\Xi(x)$ *are nuclear for any* $x \in W_L$*. As a consequence, the local algebras* $\mathcal{A}(\mathcal{O})$ *(3.13) are isomorphic to the hyperfinite type* III_1 *factor for any double cone* $\mathcal{O}$*.*

In particular, every local algebra $\mathcal{A}(\mathcal{O})$ *has cyclic vectors and therefore contains non-trivial operators.*

4. Conclusions

We have verified the modular nuclearity condition in a factorizing model with non-trivial S-matrix, thus realizing the algebraic construction of such theories in a first example. The work presented in this paper is another step in the constructive program initiated by Schroer [25, 27] which provides a further test of the utility of the modular nuclearity condition in this context. Applying the results from [10], the local algebras in this model were shown to be non-trivial and satisfy all postulates of nets of local observables. One may therefore expect that this algebraic approach is also viable in models with a more general factorizing S-matrix. The nuclearity properties of such theories will be discussed elsewhere.

In comparison to other treatments of factorizing models [2, 3, 21], it is apparent that the approach described here and the formfactor program are complementary to each other: whereas explicit constructions of n-point functions and computations of formfactors are more convenient in the field-theoretic context, the algebraic approach appears to be better suited for the discussion of existence problems. It seems that one needs both approaches for a proper understanding of this area of quantum field theory.

In conclusion, we mention that the methods developed here can also be applied to theories of free fermions in order to establish nuclearity properties. Consider a free Fermi field on $(d+1)$-dimensional Minkowski space, and a double cone $\mathcal{O}_r := \{(x_0, x) \in \mathbb{R}^{1+d} : |x_0| + |x| < r\}$. Let $\mathcal{A}(\mathcal{O}_r)$ denote the corresponding von Neumann algebra of observables localized in $\mathcal{O}_r$. Fixing a Hamiltonian H as the generator of translations along some timelike direction, we consider the maps

$$\Theta_{\beta,r} : \mathcal{A}(\mathcal{O}_r) \longrightarrow \mathcal{H}, \qquad \Theta_{\beta,r}(A) := \mathrm{e}^{-\beta H} A\Omega, \qquad \beta > 0. \tag{4.1}$$

Analogous to $\Xi(x)$ (3.14), $\Theta_{\beta,r}$ can be formulated in terms of a real subspace of the one-particle space [11]. Setting $X = \mathrm{e}^{-\beta H}$, the mathematical structure described in section 2 is thus seen to be present also in this context. Hence proposition 2.1 can be applied to reduce the nuclearity problem for $\Theta_{\beta,r}$ to a question on the one-particle space, which is known to have an affirmative answer [11]. Applying the calculations from the appendix of [11], one can also derive bounds on the nuclear norms $\|\Theta_{\beta,r}\|_1$ in terms of β, the spacetime dimension and the diameter r of the localization region considered, thereby establishing the energy nuclearity condition and all of its consequences [8, 11] in this class of models.

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